Minimal grand unification model in an anthropic landscape

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Abstract. It has been recently pointed out by Arkani-Hamed and Dimopoulos that if the universe is a landscape of vacua, and if therefore fine-tuning is not a valid guidance principle for searching for physics beyond the standard model, supersymmetric unification only requires the fermionic superpartners. We argue that in that landscape scenario, the fermionic superpartners are not needed for unification, which can be achieved in SO(10) either via a direct breaking to the standard model at the grand unification scale or through an intermediate gauge symmetry. In most minimal SO(10) models, the proton lifetime is long enough to avoid the experimental bounds. These models are the truly minimal fine-tuned extensions of the standard model in the sense proposed by Davoudiasl et al.

1 Introduction

It has been pointed out a few years ago that the fine-tuning problem of the standard model, i.e. why the Higgs boson's mass is so small, and thus stable with respect to radiative corrections, in comparison to the Planck scale (this problem is also referred to as the naturalness problem [1]), could be explained by the anthropic principle [2]. The non-vanishing value of the cosmological constant can be seen as a failure of the fine-tuning problem as a guidance for physics beyond the standard model. Indeed if for example a symmetry, e.g. supersymmetry, was to explain the magnitude of the cosmological constant it would require a breakdown of the physical theories we are so familiar with at a scale of the order of 10^{-3} eV, and new physics would probably have been observed already.

This could imply that fine-tuning is not a valid physical question and that indeed as in any renormalizable theory the gauge and Yukawa couplings are parameters that have to be measured and whose magnitudes large or small cannot be explained from first principles. In that case it makes little sense to discuss whether a certain value of a given parameter is natural or not. The remaining problem is then to understand the splitting between the Planck or grand unified scale and the weak scale, the so-called gauge hierarchy problem. This is the approach that has been advertized in [3] where it was shown that a seesaw mechanism in the Higgs sector of a simple extension of the standard model can explain the magnitude of the electroweak scale and also trigger the Higgs mechanism. Accidental cancellations of radiative corrections involving large fine-tuning are also conceivable [4]. The discovery at the LHC of a single Higgs boson and the lack of any new physics signal would confirm the cosmological constant hint that the

fine-tuning issue is irrelevant. We nevertheless note that a fat Higgs model [5] or a composite Higgs model, see e.g. [6,7], could be a valid alternative to the standard model if only one Higgs boson was discovered at the LHC.

On the other hand one could argue that the fine-tuning issue is not irrelevant but was badly formulated. It has been recently discovered that string/M theory has a landscape of vacua [8]. At first sight this sounds like a disaster for the leading candidate for a theory of everything, but if one accepts the anthropic principle as a valid scientific explanation this abundance of vacua allows one to rephrase the fine-tuning problem. Basically the question becomes: given the probability distribution of the physical parameters of a theory, what is the probability that we happen to live in a universe, or vacuum, that has a given set of parameters and how probable is that vacuum? This question makes sense if indeed no physical principle is found in string/M theory to select a particular vacuum and if indeed string/M theory is the correct theory of nature. It should be noted that a cosmological constant of the right order of magnitude had been predicted by Weinberg [9] already a long time ago using anthropic considerations. Recently such considerations have been applied to the supersymmetry breaking scale [10].

This reformulation of the fine-tuning problem was the motivation for Arkani-Hamed and Dimopoulos [11] to propose that supersymmetry might not be required to explain a light Higgs boson but can be useful to explain the unification of the gauge couplings; see also [12]. The basic idea is to break supersymmetry at a very high scale, thereby giving large masses to all the scalar fields with the exception of the standard model Higgs boson whose mass is assumed to remain light and thus has to be fine-tuned. The new fermions appearing in the supersymmetric extension of the standard model are assumed to remain light

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enough so that they can contribute, as usually assumed, to the running of the gauge couplings and guarantee the unification of the couplings. It should be emphasized that in this framework, unification does not require an additional fine-tuning since the fermion masses could be protected by chiral symmetries. While this scenario is interesting from the phenomenological perspective (it predicts a plethora of new phenomena at the LHC), it is clearly not a minimal extension of the standard model that leads to the unification of the gauge couplings.

Motivated by this study and the minimal extension of the standard model proposed by Davoudiasl et al. [13], we consider the minimal grand unified theory in an anthropic landscape scenario. We point out that supersymmetry, or in other words the occurrence of new fermions in that scenario, is not required for grand unification. In the remaining of this work, we will describe the truly minimal grand unified model that accounts for all the, to date, observed phenomena. The simplest viable model is based on a SO(10) group [14]. We will assume that fine-tuning is a non-valid physical question or rather that it should be rephrased in terms of the landscape problematic. One should nevertheless keep in mind that the anthropic principle could apply only to certain parameters of the theory, e.g. the cosmological constant. It the sequel we will ignore this possibility and assume that fine-tuning is acceptable for all the parameters of the model.

2 Fine-tuned minimal SO(10) grand unification

We will be looking for a unified model with the following properties:
(a) numerical unification of the gauge couplings,
(b) enough free parameters to fit the fermion masses,
(c) baryogenesis,
(d) long lived proton to avoid the experimental bounds.

The dark matter, the inflaton, the cosmological constant and gravity are considered to be different sectors in the spirit of [13] and we shall not try to unify these with the remaining interactions. These sectors of the theory are assumed to be described by the minimal models presented in [13]. We will argue that non-supersymmetric SO(10) grand unified models are viable candidates for grand unification. It should be nevertheless mentioned, that on the contrary to supersymmetric theory, there is no obvious dark matter candidate in these models. Obviously one can easily either introduce a singlet under SO(10) to describe dark matter or introduce some scalar multiplet in a representation of SO(10) that does not spoil the unification of the standard model gauge couplings.

There are different ways to break the grand unified group SO(10) to the standard model group SU(3) × SU(2) × U(1); we shall present four different non-supersymmetric models that are phenomenologically viable in the sense that the proton lifetime is long enough to escape discovery and that the gauge couplings of the standard model unify. Obviously there are many ways to

satisfy the proton decay constraint while satisfying the unification constraints, and we will give two examples of SO(10) models that fulfill these requirements.

Baryogenesis in non-supersymmetric SO(10) grand unified theories can happen at the grand unification scale [15] as one of the predictions of the model is that the baryon number is violated. But a potentially serious problem with that scenario is the low value of the thermalization temperature of the inflaton. The unification scale is expected to be of the order of 10^{15} GeV. Heavy Higgs bosons and gauge bosons leading to baryon decay are expected to have a mass of the order of the grand unification scale to avoid problems with the bounds on proton decay. They thus have a mass greater than that of the inflaton and it seems kinetically impossible to produce them directly through inflaton decay. However one can imagine a scenario where grand unified gauge and Higgs bosons leading to baryon decay are produced nonthermally [16]. We shall nevertheless invoke leptogenesis [17] as our mechanism to generate the baryon asymmetry as it seems more plausible and appears automatically in the minimal SO(10) model.

As mentioned previously, leptogenesis is not a requirement, but one has to go beyond the **10** to get a realistic spectrum for the fermion masses. One can introduce for example the **126**. If D parity (a Z_2 discrete symmetry contained in SO(10)) is broken at the grand unified scale, the seesaw formula appears naturally. Note that the seesaw mechanism is not imposed to explain the smallness of the neutrino masses or leptogenesis, but it just follows from the minimalistic assumption. The **126** generates Majorana masses for the neutrinos, and baryogenesis happens through leptogenesis.

2.1 Minimal SO(10) grand unification model

The first model is that proposed by Lavoura and Wolfenstein [18]. It is the minimal SO(10) grand unification model broken directly at the grand unification scale to the standard model gauge group. The fermion masses are generated by the Higgs multiplets in the **10**, **126** and **210** representations. The Higgs bosons in the **10** and **126** representations break SO(10) to $SU(2)_L \times SU(2)_R \times SU(4)_{PS}$. The (2,2,1) of the **10** gets a vacuum expectation value v_1, the (1,3,10) of the **126** gets a vacuum expectation value v_R, the (2,2,15) of the **126** gets a vacuum expectation value v_{15} with $v_{15} < v_R$ and the (1,1,1) of the **210** gets a vacuum expectation value v_U with $v_U = \mathcal{O}(M_U)$, where M_U is the grand unification scale. Obviously one has to impose $v_U > v_R$. These are the requirements to fit the fermion masses and to unify the gauge couplings at a scale M_U; see [18] for details.

Lavoura and Wolfenstein have shown that if the heavy Higgs bosons and heavy gauge bosons have masses smaller (e.g. a factor 30) than the energy scale where the gauge symmetry is broken, then the running of the gauge couplings can be significantly affected and grand unification is possible for a range of parameters. The mechanism proposed in [18] is based on the observation that there are

nine gauge bosons in SO(10) that are not standard model gauge bosons but that do not lead to proton decay which are assumed to have masses of the order of $v_R = M_R$. The main idea is to use these gauge bosons that do not lead to proton decay to affect the running of the gauge couplings significantly, whereas the remaining non-standard model gauge bosons of SO(10) that lead to proton decay are assumed to have a mass of the order of the grand unification scale M_U.

To illustrate our point, we shall use the one loop results derived in [18]:

$$\ln \frac{M_U}{M_Z} \tag{1}$$
$$= \frac{1}{128}\left[60\pi\omega_1(M_Z) - 60\pi\omega_2(M_Z) + 5(\lambda_1 - \lambda_2)\right],$$

$$\omega_G(M_U) \tag{2}$$
$$= \frac{1}{128}\left[95\omega_1(M_Z) + 123\omega_2(M_Z) + \frac{95\lambda_1 + 123\lambda_2}{12\pi}\right],$$

$$\omega_3(M_Z) \tag{3}$$
$$= \frac{1}{128}\left[-115\omega_1(M_Z) + 333\omega_2(M_Z) \right.$$
$$\left. + \frac{-115\lambda_1 + 333\lambda_2 - 218\lambda_3}{12\pi}\right],$$

where ω_1 is the inverse of the U(1) gauge coupling, ω_2 is the inverse of the SU(2) gauge coupling, ω_3 is the inverse of the SU(3) gauge coupling and ω_G is the inverse of the SO(10) gauge coupling. The λ's represent the contributions of the heavy gauge bosons and Higgs bosons to the running of the gauge couplings and are given by [18]

$$\lambda_1 = 8 + \frac{294}{5}\ln\frac{M_U}{M_R} - \frac{274}{5}\ln\frac{M_U}{M_1} - \frac{142}{5}\ln\frac{M_U}{M_2}$$
$$- \frac{36}{5}\ln\frac{M_U}{M_3} - \frac{114}{5}\ln\frac{M_U}{M_4} - \frac{24}{5}\ln\frac{M_U}{M_5}, \tag{4}$$

$$\lambda_2 = 6 - 50\ln\frac{M_U}{M_1} - 40\ln\frac{M_U}{M_3} - 30\ln\frac{M_U}{M_5}, \tag{5}$$

$$\lambda_3 = 5 + 21\ln\frac{M_U}{M_R} - 62\ln\frac{M_U}{M_1} - 17\ln\frac{M_U}{M_2}$$
$$- \frac{36}{5}\ln\frac{M_U}{M_3} - 12\ln\frac{M_U}{M_4} - 12\ln\frac{M_U}{M_5}, \tag{6}$$

where M_U is the grand unification scale, M_R is the mass of the nine gauge bosons of SO(10) not contained in the standard model and that do not lead to proton decay, M_1 is the mass of the scalars in the (1,1,6) and (2,2,15) contained in the **126**, M_2 is the mass of the scalars contained in the (1,3,10) contained in the **126**, M_3 is the mass of the scalars contained in the (1,3,10) contained in the **126**, M_4 is the mass of the scalars in the (1,3,15) contained in the **210** (they have a negative impact on SO(10) unification and one thus expect $M_U/M_4 \sim 1$), M_5 is the mass of the scalars in the (3,1,15) contained in the **210** (they are beneficial for SO(10) unification). Note that there is a typographical mistake in (17) of [18], a factor $-24/5\ln(M_U/M_5)$ is missing in the definition of λ_1^S.

For the proton lifetime estimate, we shall use

$$\tau_{p\to e^+\pi^0} = \frac{5}{8}\left(\frac{\alpha_U^{\mathrm{SU(5)}}}{\alpha_U^{\mathrm{SU(10)}}}\right)^2 \tag{7}$$
$$\times 4.5 \times 10^{29}\left(\frac{M_U}{2.1 \times 10^{14}\,\mathrm{GeV}}\right)^4\,\mathrm{yr},$$

following Lee et al. [21] and assuming that $\alpha_U^{\mathrm{SU(5)}} \approx \alpha_U^{\mathrm{SU(10)}}$. For a numerical estimate we use $\omega_1(M_Z) = 1/0.016887$, $\omega_2(M_Z) = 1/0.03322$ and find that the set of parameters $M_1 = M_2 = M_3 = M_4 = M_5 = M_U$, $M_R = 1/104 M_U$ lead to $\alpha_s(M_Z) = 0.120$, $M_U = 4.3 \times 10^{15}$ GeV and $\tau_{\mathrm{proton}} = 4.4 \times 10^{34}$ yr. Another set of parameters is e.g. $M_1 = M_2 = M_4 = M_U$, $M_R = 1/38 M_U$, $M_3 = 1/2.18 M_U$ and $M_5 = 1/2 M_U$, leading to $\alpha_s(M_Z) = 0.120$, $M_U = 2.95 \times 10^{15}$ GeV and $\tau_{\mathrm{proton}} = 1 \times 10^{34}$ yr. It seems difficult to push the proton lifetime above 10^{34} yr. This is a prediction of that model with direct breaking to the standard model at the grand unification scale. Besides neutrino masses, the only new phenomenon is proton decay with a lifetime of the order of 10^{34} yr. Note that this is only one order of magnitude above the present experimental limit for proton decay [19]. It is nevertheless possible to have a longer proton lifetime if there is an intermediate scale [20–22].

2.2 Minimal models with two steps breaking of SO(10)

Mohapatra and collaborators have studied these cases extensively. Four different breaking schemes can be considered:
(a) SO(10) → G_{224D} = SU(2)$_L$ × SU(2)$_R$ × SU(4)$_C$ × D;
(b) SO(10) → G_{224D} = SU(2)$_L$ × SU(2)$_R$ × SU(4)$_C$;
(c) SO(10) → G_{2213D} = SU(2)$_L$ × SU(2)$_R$ × U(1)$_{B-L}$ × SU(3)$_C$ × D;
(d) SO(10) → G_{2213} = SU(2)$_L$ × SU(2)$_R$ × U(1)$_{B-L}$ × SU(3)$_C$, assuming that the intermediate scale M_I is where SO(10) is broken. Case (a) arises if the Higgs multiplet used is in the **54**. Cases (b) and (c) arise if a Higgs multiplet in the **210** is used. The nature of the intermediate gauge symmetry depends on the details of the Higgs potential. Finally case (d) arises if a **45** and a **54** are used to break SO(10). In all cases a **126** and a **10** are needed to break the intermediate gauge symmetry to U(1)$_{em}$ of QED. The predictions of each of these models for the proton lifetime are [21]
(a) $\tau_{p\to e^+\pi^0} \sim 1.44 \times 10^{32}$ yr,
(b) $\tau_{p\to e^+\pi^0} \sim 1.44 \times 10^{37.4}$ yr,
(c) $\tau_{p\to e^+\pi^0} \sim 1.44 \times 10^{34.2}$ yr,
(d) $\tau_{p\to e^+\pi^0} \sim 1.44 \times 10^{37.7}$ yr.
The uncertainties in these predictions have been discussed in [21,22]. Despite these uncertainties, model (a) is probably excluded by direct searches for proton decay. A CP violating phase in the CKM matrix compatible with present experiments requires another multiplet e.g. a **120** [23]. This multiplet is assumed to be very heavy, i.e. its mass is of the order of the grand unification scale, such

that it does not contribute to the running of the gauge coupling.

2.3 Predictions of SO(10) grand unified models

The predictions of SO(10) grand unified models are
(a) neutrino masses and oscillations are expected,
(b) proton decay, the lifetime of the proton is around 10^{34} yr if SO(10) is broken directly to the standard model at the GUT scale or up to about 10^{38} yr if there is an intermediate scale at 10^{13} GeV,
(c) one light Higgs boson will be observed at the LHC but no signal for any new physics whatsoever.

These are the three firm predictions of a SO(10) grand unified theory which is either directly broken at the grand unification scale to the standard model or which is broken first to a subgroup and then at an intermediate scale to the standard model.

It should be noted that although we are giving up to explain the naturalness or fine-tuning problem, the gauge hierarchy problem can be understood in the framework of a grand unified theory. A renormalization group equation "explains" the hierarchy problem: once the high scale value is fine-tuned, the low scale value of the Higgs boson expectation value can be predicted.

Within string/M theory, where the landscape reasoning to solve the fine-tuning problem makes sense, gauge couplings are expectation values of moduli and can thus have a time dependence. As pointed out in [24], if gauge couplings have a time dependence, one might be able to obtain some information on the nature of the grand unified theory. Although this prediction of these models is more speculative than the three predictions mentioned above, an observation of a time dependence of the gauge couplings fulfilling the relations derived in [24] together with the observation of only one Higgs boson at the LHC would have to be interpreted as a hint that the landscape scenario is a reasonable explanation for the fine-tuning problem of the standard model.

3 Conclusions

It is important to emphasize the fact that there are many ways to satisfy the proton decay constraint while satisfying the unification constraints, we have given two examples of SO(10) models that fulfill these requirements.

If the world we live in is indeed fine-tuned, grand unification does not require supersymmetry. Supersymmetry might still be necessary for quantum gravity, but there is no good motivation to require that any superpartner has a mass below the Planck scale. It might thus be a hopeless task to detect any effect of supersymmetry. On the other hand, proton decay is unavoidable and is a clear signature of a grand unification. One of the predictions of SO(10) neutrino masses and oscillations has already been observed. We have presented the minimal models, one could imagine decoupling the different scales of the models and introducing more scalar multiplets. It is interesting to note

that, once fine-tuning is allowed, the only motivation for supersymmetric unification is dark matter. Minimal grand unified non-supersymmetric models do not have "natural" candidates. But, without any further experimental evidence, dark matter remains a very weak motivation for low energy supersymmetry or split supersymmetry. We note that more complicated non-supersymmetric models often have dark matter candidates. For example, in technicolor models the lightest technibaryon could be stable and be the dark matter. An interesting possibility is that nature is indeed supersymmetric at the grand unification scale and that there is a nearly exact chiral symmetry that protects the supersymmetric dark matter candidate from developing a very large mass, but that on the other hand the remaining fermionic superpartners are very massive because their chiral symmetries are more strongly broken. In that scenario, supersymmetry would only be required to explain dark matter.

The LHC might just discover one single Higgs boson; this would be a second piece of evidence, after the cosmological constant, that the guidance principle we had for model building was not the right one. This could be explained by the anthropic principle, if we live in a landscape of vacua, or simply by the fact that renormalization is a physical principle and that gauge and Yukawa couplings are just parameters of the theory that have to be measured. As such their magnitudes, small or large, do not need to be explained. The only physical question that remains is to explain the splitting between the Planck scale and the weak scale (gauge hierarchy problem), but this seems rather simple to understand within the framework of a grand unified theory, as it would be the consequence of gauge symmetry breaking and the running of the parameters of the Higgs boson's potential from the grand unified scale to the weak scale. Another interesting challenge is to understand how to generate or trigger the weak phase transition. This is naturally explained in supersymmetric models, but at the price of supersymmetry breaking. As a conclusion, we want to emphasize that fine-tuning as a guidance principle for searching for physics beyond the standard model might not be the right one for different reasons and one should remain very open minded when it comes to analyzing the LHC data, as a complete surprise is not that improbable.

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